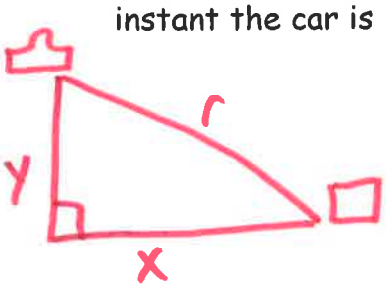
$V_{cone} = \frac{\pi}{3} r^2 h$ $V_{sphere} = \frac{4}{3}\pi r^3$ $SA_{sphere} = 4\pi r^2$

Calculus 1 : Related Rates (B) Name(s) _____

5) A car is traveling toward an intersection at a rate of 88 feet per second and a bus is traveling at a rate of 62 feet per second away from the intersection (the car and bus are perpendicular to each other). How fast is the distance between two vehicles changing at the instant the car is 155 feet from the intersection and the bus is 80 feet away?



$\frac{dy}{dt} = -88$ ft/sec $\frac{dr}{dt} = ?$ $y = 155$ $x = 80$ $80^2 + 155^2 = r^2$

$\frac{dx}{dt} = 62$ ft/sec $r = 174.428$ ft.

$x^2 + y^2 = r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

$2 \cdot 80 \cdot 62 + 2 \cdot 155(-88) = 2 \cdot (174.428)\frac{dr}{dt}$

$9920 - 27{,}280 = 348.856\, dr/dt$

$-17360 = 348.856\, dr/dt$

$\boxed{\frac{dr}{dt} = -49.763 \text{ ft/sec}}$

6) A spherical snowball is rolling down a hill and its size is increasing at a rate of 7.5 cubic inches per minute. Find the rate at which the surface area is changing at the instant the diameter is 13 inches.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$7.5 = 4\pi \cdot 6.5^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = .014$ in/min

$SA = 4\pi r^2$

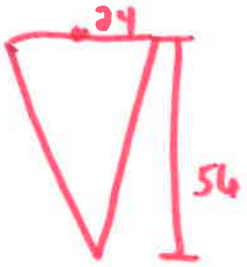
$\frac{dSA}{dt} = 8\pi r \frac{dr}{dt}$

$= 8 \cdot \pi \cdot 6.5 \cdot .014$

$\boxed{\frac{dSA}{dt} = 2.297 \text{ in}^2/\text{min}}$

$V_{cone} = \frac{\pi}{3} r^2 h$ $V_{sphere} = \frac{4}{3}\pi r^3$ $SA_{sphere} = 4\pi r^2$

7) A water tank in the form of an inverted cone is being emptied at a rate of 11 cubic meters per minute. The height of the cone is 56 meters and the radius of the top is 24 meters. Find the rate of change of the radius of the water level at the instant the water is 14 meters deep.



$\frac{dV}{dt} = -11 \text{ m}^3/\text{min}$

$\frac{dr}{dt} = ?$ when $h = 14$

$\frac{24}{r} = \frac{56}{h}$

$24h = 56r \rightarrow h = \frac{56r}{24} = \frac{7}{3}r$

$r = \frac{24}{56}h$

$r = \frac{3}{7}(14)$

$r = 6$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3} \cdot r^2 \cdot \frac{7}{3} r$

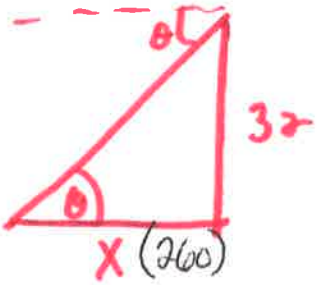
$V = \frac{7\pi}{9} r^3$

$\frac{dV}{dt} = \frac{7\pi}{3} r^2 \frac{dr}{dt}$

$-11 = \frac{7\pi}{3} \cdot 6^2 \frac{dr}{dt}$

$\frac{dr}{dt} = -.042 \text{ m/min}$

8) A police officer is standing on top of a highway overpass that is 32 feet above the highway. A car traveling at 95 feet per second passes under the overpass. What is the rate of change of the angle of depression of the police officer at the moment the car has traveled 260 feet beyond the overpass?



$\frac{dx}{dt} = 95 \text{ ft/sec}$

$\frac{d\theta}{dt} = ?$ when $x = 260$

$\tan\theta = \frac{32}{260}$

$\theta = .122 \text{ rad.}$

$\tan\theta = \frac{32}{x}$

$\tan\theta = 32x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -32x^{-2} \frac{dx}{dt}$

$\sec^2(.122) \cdot \frac{d\theta}{dt} = \frac{-32}{260^2} \cdot 95$

$\frac{d\theta}{dt} = -.044 \text{ rad/sec}$